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ABSTRACT

The Nested Spiral Approach (NSA) is an integrated instructional approach used to promote the motivated learning of mathematics problem solving in limited-English-proficient (LEP) students. The NSA is described and a trial use is discussed. The approach extends, elaborates, and supplements existing education and instruction theories to help LEP students by paying special attention to student needs and incorporating strengths stemming from their cultural heritage into teaching practices. NSA activates and uses interactive characteristics in four aspects of learning through: balanced emphasis on native language and English; three teaching cycles in each lesson; five steps in each cycle; and the application of six guiding principles. The principles include the following: (1) connection, emphasizing interaction between student and teacher, between teaching and learning, and among mathematical topics; (2) continuity of learning activities; (3) comparison for sharpening contrast and making analogies between cultures, languages, and mathematics topics; (4) complementation, which acknowledges and uses the interdependence between content and language; (5) flexibility in learning context; and (6) versatility through exposure to various learning activities. Trial of this approach with a class of bilingual Chinese eighth-graders showed encouraging results. All students made significant progress in both mathematics and language learning. (Author/MSE)

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Teaching Mathematics Problem Solving to Students with Limited English Proficiency through Nested Spiral Approach

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TEACHING PROBLEM SOLVING TO LEP STUDENTS THRU NSA

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Abstract

The purpose of this paper is to report an integrated instruction approach, *Nested Spiral Approach* (NSA), which has been found to be effective in promoting motivated learning of mathematics problem solving by students with *limited English proficiency* (LEP). This approach extends, elaborates, and supplements existing education and instruction theories to help LEP students better learn mathematics by paying special attention to their needs and by incorporating their inherent strength (stemmed from cultural heritage) into actual teaching practice.

NSA activates and utilizes the interactive characteristics among four aspects of learning and puts them into proper perspective through: (a) balanced emphasis on both languages, (b) three teaching cycles in each lesson, (c) five steps in each cycle, (d) tactic applications of six guiding principles.

Preliminary trial of this approach showed very encouraging results: All students made significant progress in mathematics and languages alike. This, in turn, can further facilitate successful learning. As a result, a cycle of well-motivated learning is in motion, geared toward achievement not only in the cognitive domain but also in the affective domain.

Teaching Mathematics Problem Solving to Students with Limited English Proficiency through Nested Spiral Approach

Cultural differences and language barrier present difficulties to students with *limited English proficiency* (LEP). These students are primarily new immigrants in the U.S. with a relatively short period of residence. Adjustment to the new and different environment here is not easy at all. In order to alleviate the heavy pressure sustained by them, many bilingual programs have been created. In implementing these programs, individual teachers have employed their own teaching strategies and approaches. In an effort to devise an effective instructional approach for Chinese LEP students in learning mathematics problem solving, I came up with the *Nested Spiral Approach* (NSA) and tried it out with my class. It seems that the strategies, on the basis of this approach, worked very well. Moreover, upon scrutinizing the fundamental framework and principles, I discovered that this approach could give some insight regarding teaching mathematics to students of other ethnic backgrounds.

The general rationale is that cultures and languages are an inseparable part of our real life and there exist strong ties between real life experiences and mathematics. Specifically, the nested spiral approach recognizes and makes use of two factors of fundamental importance. One of them has to do with the fact that LEP students possess strengths stemming from their cultural and linguistic heritage. However, these strengths are seldom exploited, not to mention utilized. In this connection,

Cummins (1981) made a statement that has profound significance: "One of the main reasons for the success of quality bilingual programs is that they encourage minority students (and probably the minority community) to take pride in their cultural background" (p. 41). This explains, in a nutshell, the key issue in bilingual education. Viewed in the context of the interplay between cognitive growth and affective development for the students, we can fully appreciate the implications of cultural and linguistic heritage.

The other factor of fundamental importance is intimately related to the first one, except that it is at a different level, with more detailed structure in apparent action. NSA recognizes and utilizes the interaction among four aspects of learning comprising mathematics content, cultures, languages, and real life experiences, as well as interaction between teacher and students. As a result, it enhances motivation and interest, thereby promoting meaningful learning in a natural way. Moreover, its continual application sets up a cycle of spontaneous learning through two interrelated elements. On one hand, because of deep interest, the students want to learn more. On the other, the more they learn, the better can they learn and vice versa. In other words, NSA provides the students with a favorable learning inducing atmosphere in which a snowball rolling process of productive and pleasant learning is going on. This is the spirit of NSA.

NSA is based on four premises: (a) Bruner's instruction theory, (b) Cummins' language proficiency concepts in bilingual education theory, (c) Polya' theory of problem solving, and (d) the standards and practices of the National Council of Teachers of Mathematics (NCTM). It is essentially

an elaboration and application of these educational theories and practices with innovation by paying special attention to LEP students' needs and by incorporating their inherent strength (stemmed from cultural heritage) into actual teaching practice.

Bruner (1966) delineated four major features of an instructional theory after asserting its needs for being prescriptive and normative. The first feature is to specify the learning experiences for fostering a favorable disposition toward learning. The second is to organize the knowledge to facilitate understanding. The third is to stipulate the optimal sequence of learning activities. The fourth is to describe the nature and pacing of reinforcement. We can see that he stressed the prominent importance of cultural, motivational and personal factors affecting the desire to learn and the effectiveness of teaching. This has great significance concerning academic endeavor in the context of bilingual education. Moreover, Bruner realized the importance of a sequencing strategy.

Reigeluth and Curtis (1987) commented on Bruner's spiral curriculum in the context of sequencing strategies by praising Bruner's ideas and indicating the need for development of guidance for creating a spiral curriculum. NSA aims at such a development for teaching mathematics problem solving to LEP students. In fact, NSA is a generalization of the classical spiral approach to multidimensions encompassing cultures, languages and real life experiences as well as subject contents.

Saville-Troike (1979) expressed the importance of the relationship between language and culture, as well as between education and these two

elements in terms of the influence of culture on language, on a person's self-concept, and other aspects of affective domain.

In recent research on general bilingual education, the prevailing basic concept is *Common Underlying Proficiency* (CUP). This model of bilingual proficiency states that there is a common fundamental underlying base for the proficiency development of two languages in a person, provided that adequate motivation and exposure to these languages are given. In other words, learning two languages, first language (L_1) and second language (L_2), simultaneously is helpful to the acquisition of L_2, which is English for LEP students. Also, a clear distinction should be made between everyday conversational language skills and academically oriented language abilities. The former is termed *Basic Interpersonal Communication Skill* (BICS), whereas the latter is called *Cognitive Academic Language Proficiency* (CALP). Both BICS and CALP are necessary for successful schooling. However, in the pursuit of academic endeavors, CALP is primarily what is required of the students. Additionally, CALP encompasses the highly academic aspects of linguistic skills of both L_1 and L_2. These aspects are manifestations of a common underlying foundation (Cummins, 1981). Thus, technical English vocabulary may be a key to effective learning by LEP students. NSA recognizes, and utilizes this fact frequently, including its implementation during the first cycle of the approach, as can be seen from Table 1.

Polya (1973) promulgated that the understanding of a problem is the first step toward a solution of any problem. Thus, for LEP students, the

importance of learning technical vocabulary in English is indeed fundamental. This is why Table 1 enlists the essential elements needed for students to master at different levels of progress and exhibits the correlation between them. Note that technical vocabulary in L_2 is only a part of the requirements.

With the advent of NCTM's recommendations (1980, 1983), problem solving became the center of the mathematics curriculum for secondary school mathematics. Consequently, real life experience serves not only as a passive source of material for instruction, but also an effective active stimulant for promoting interest and cultivating favorable disposition toward learning. This has special meaning in the context of mathematics problem solving on the part of LEP students because culture and real life experience are closely linked together.

The purposes of the original nested spiral approach study were twofold: (1) to develop curricular units designed to enhance the algebraic problem solving competency of bilingual (Chinese) eighth grade students and (2) to conduct a preliminary trial of the units to determine their effectiveness with the target population. These units would attempt to exploit students' linguistic and cultural backgrounds to improve problem solving performance through the use of NSA.

The Nested Spiral Approach

The Foundations

NSA activates and utilizes the interactive characteristics among four aspects of effective and meaningful learning, namely: mathematics contents, real life situations, culture, and languages as implied by the educational theories and practices mentioned above. Moreover, it puts these elements in proper perspective through balanced emphasis on both languages in three teaching cycles, five steps in each cycle. This will not only ensure continuity between steps and connection between topics, but also reveal contrast and complementation between the two languages as well as between mathematics and languages. As such, NSA is a concrete and dynamic mechanism for carrying out instruction procedure. It also serves as a set of guidelines for developing the needed curriculum when viewed from another angle.

The Guiding Principles and Dynamic Interactions

The nested spiral approach presented here is an integrated teaching approach encompassing the four aspects of learning as mentioned above, with balanced emphasis on both languages, in an interactive manner. To guide its implementation, to activate the dynamic interaction among the various aspects of learning as well as between teacher and students, the following six principles are used. They are all equally important and related to each other. However, for convenience of presentation, I am introducing them in the order below. First of all, the principle of connection emphasizes the interaction between teacher and students, among the various aspects of

learning, as well as the interrelationships among topics of mathematics. It lays out the fundamental framework of NSA with the main elements in proper perspectives. As such, it serves the purpose of an organizer in general, and a "horizontal plan" in particular. Second, the principle of continuity stresses the need for continuity of learning activities and the steps for carrying them out. Another way of describing it would be as follows. It calls for paying special attention to maintain the continuity between steps, between cycles in the instruction process. In an effort to keep abreast with the terminology used in explaining the principle of connection, we can call the principle of continuity as the "vertical view" of NSA.

Having introduced the horizontal and vertical frames, we are ready to describe the principle of comparison. This principle dictates the contributions of "comparison" to the establishment of sharpening the contrast, illuminating the analogy between different cultures, languages, or topics of mathematics for enhancing motivated learning. We note that both the similarities and the differences of the elements compared are singled out for facilitating learning. Next, the principle of complementation recognizes and makes use of the implications of CUP and CALP concepts and the nature of interdependence between subject contents and languages. L_1 and L_2 together lay the foundation of academic literacy skills, with L_1 and L_2 complementing each other. Moreover, language proficiency fosters learning of mathematics contents, which in turn pay back the dividend in the form of better understanding and deeper appreciation of technically oriented terminology and associated language skills, and vice versa. Most

importantly, this cycle of helping each other continues as the learning process goes on.

Now, the principle of flexibility asserts that informal, relaxed learning atmosphere and teachers' flexible and resourceful ways to deal with instructional situations can nurture fruitful learning outcomes with ease. We can give a one-word label to this principle. The word is "atmosphere." It is indispensable to the students' growth in terms of both cognitive and affective measures, just as air is vital to their very existence.

Finally, the principle of versatility reminds us of the importance of keeping students' interest keen by exposing them to various kinds of learning activities for mastering one topic as well as for establishing relationships among various topics. A metaphor here would be that the principle of versatility plays the role of coordinator in the endeavor of applying NSA.

These principles, when applied in conjunction with the mechanism of NSA described below, can serve as a catalyst for triggering the snowball rolling process mentioned above.

A Typical Instructional Model

A typical instructional model for the proposed approach is for three levels of achievement. Each level emphasizes a set of specific objectives regarding mathematics competency and language proficiency (see Table 1).

The mastery of a lesson is achieved by using three *cycles* of teaching and learning, each one corresponding to one level. The five steps of preview, exploration, lecture, exercise, and review which form the concrete

mechanism for the approach are carried out in each cycle. An outline of the three cycles is given below:

Cycle 1: Tell stories about cultural and historical aspects of the subject, as a start, to motivate and set the tone for the lesson. Informal discussion is encouraged and can be in either language or in any style approved by the teacher. Introduce new English technical vocabulary needed for this topic in a casual manner or in the form of games. In this cycle, three elements are emphasized: (1) motivation by using students' mathematical competency in L_1 and cultural comparison, (2) gradual introduction of L_2 vocabulary, and (3) presenting basic mathematics concepts in L_1.

Cycle 2: Based on the foundation provided in Cycle 1, students are encouraged to learn more about the concepts, principles, and rules in mathematics, half in L_1 and half in L_2. In this cycle, practice in the skills in mathematics contents and simple technical English is the main theme.

Cycle 3: Apply the materials learned in Cycles 1 and 2 through the use of real life experiences with cultural contrast to provide a relaxed and yet thought-provoking atmosphere to nurture creative thinking needed for real problem solving. Activities incorporating mathematics concepts, principles, rules and language skills will be carried out in the context of daily life applications of mathematics. Teacher and students try to use L_2 as much as possible, aiming at 100 percent L_2.

Since the approach is meant to be flexible and dynamic, we could extend the three cycle and three level notion to any topic of choice, however

small or big, depending upon the need of the students as determined by the teacher. The time appropriation for the four aspects of learning varies from cycle to cycle, whether we are talking about a minute topic, a section or a unit. The interaction among mathematics contents, culture, languages, and real life experiences is always there, even though one or two of them may be more prominent than the others at one instance. This is one of the reasons why we call the proposed teaching approach the nested spiral approach.

Curriculum design and instruction go hand in hand. It is only natural that the same spirit prevails in both phases of the same project of facilitating students' learning. Therefore, the fundamental guiding principles are meant to be applied to both instruction activities and curriculum development tasks.

The Experimental Trial of the Proposed Approach

Background of Participating Students

An experimental trial of the nested spiral approach was carried out to determine its effectiveness for the target population. The sample used in this study consisted of a class of bilingual Chinese eighth graders in the New York City School System, with an equal number of students of each sex. These students exhibited limited English proficiency as a result of their recent arrival in this country and their language and cultural heritage. The language assessment battery scores provide an indication of the status of their English language proficiency. The mean score was 8th percentile. The score ranged from 1st to 15th percentile. The average period of U.S. residence was approximately one year; however, some of them had come here only four to six months prior to this trial treatment.

The Treatment Course

The treatment curriculum units including decimals, fractions, ratio and proportions, percent, and algebraic expressions were taught to them using the proposed approach. The course duration was about six weeks including a pretest and a posttest. The pretest was administered at the onset of the treatment by using a national standard test. It was given in the original English (L_2) version and a Chinese (L_1) version translated from the standard test by the author of this article, under identical conditions. The posttest, consisting of L_2 and L_1 versions similar to the pretest, was conducted at the end of the treatment to measure students' achievement during the course. Both language proficiency and mathematics competency were assessed through these tests.

The Test Results

Comparing the results of the two language versions of the pretest (Table 2), it appears that, on the average, students' scores for the L_2 version were about 67% of those for the L_1 version of the test. This seems to indicate that, due to limited English proficiency, these students did not reflect their true competency in mathematics on the L_2 version of the pretest. This was also confirmed by observing the pattern of errors made on the pretest. These errors appear to be caused by difficulty in understanding the meaning of the statements of the test problems.

The analysis of the results of the posttest followed exactly the same method and procedure as used for the pretest (see Table 2). It is interesting and informative to compare pretest and posttest results. Let us look at the

L_1 version of the tests. We can easily discover that the ratio M_3/M_1 indicates students' progress in the mastering of the mathematics contents. This value is 1.49, which means that, on the average, the students learned well in the sense of performing 49% better at the end of the treatment than at the beginning. By the same token, the ratio $M_4/M_2 = 1.75$ indicated their progress according to the results of the L_2 version of the pretest and posttest. Attention is called to the following observation. M_3/M_1 measures rather accurately the students' progress in the learning of mathematics, whereas M_4/M_2 signified the combined, overall progress in both mathematics contents learning and English language acquisition.

Conclusions and Recommendations

Conclusions

As indicated above, the students progressed both in English language proficiency and mathematics competency after being taught by the nested spiral approach. This progress could be measured either in terms of any individual students or a whole group. Moreover, the students' attitude towards learning of mathematics and English, especially those language skills closely related to mathematics, became much more positive than before. They actively participated in group learning activities as well as individual exploration, all with zest and motivation. Self confidence has improved tremendously as a result of this new type of learning experience. Most of them liked this so much that they wanted to do more. It appeared that a long-lasting motion of well motivated learning in cycles is in fact activating. Therefore, the nested spiral approach enhanced their learning of

mathematics problem solving, with better performance and growth both in the cognitive domain and in the affective domain.

Suggested Future Work

Future extension of the present study could proceed in several directions: The proposed teaching approach could be applied to the teaching of LEP students at high school level or some other levels with the same ethnic background. If we call this an expansion of the scope of applicability of NSA in the vertical direction, then the natural thing to do is to see whether we could expand that scope horizontally. Thus, we have the next direction for future probing. We see that, due to similarities among various ethnic groups, in terms of common difficulties and problems (to a certain extent at least), the guidelines for establishing a set of useful teaching strategies could be comparable. In other words, the present instructional model might be able to serve as a model in devising instructional approaches appropriate for LEP students from other ethnic groups.

Next, because generalization is a virtue advocated by us dedicated educators, I am taking this opportunity to emphasize the point that culture and languages are inseparable parts of our real life. Then the logical conclusion is that the nested spiral approach might shed some light on the search for better teaching strategies and instructional approaches to be used in teaching some other subject areas as well. At the time of this writing, this is only a hypothesis that warrants testing and verification in the near future. Let us pause for a moment and compare mathematics and science teaching

and observe the connections. It is well known that mathematics and science go hand in hand primarily because of the "bridge," namely problem solving, that connects them. In view of this, it is conceivable to apply NSA to teach LEP students science with success, provided certain needed modifications (to the present approach) are made. Along the same line, we note that the subject of social studies is deeply rooted in cultures and is closely related to real life and languages. Consequently, this subject could also be taught with zest and production by applying NSA with some necessary fine tunings which are unique to the subject area.

Finally, an empirical study employing statistical methods can be conducted when a large sample is available. The study could be associated with each of the possible theoretical endeavors mentioned above. It is noted, in passing, that the experimental trial reported in this paper is only a probing, pilot study. Its large sample and extensive scope counterpart should be of great interest and value. The undertaking of such a project is awaiting interested persons in the near future.

Table 1

Objectives at Three Levels

	Mathematics Contents	English Language Proficiency
I.	Concepts	Vocabulary
II.	Principles, rules	Grammar, sentence making
III.	Applications	Mathematics text comprehension

Table 2

Test Results

		Pretest	Posttest
L_1 Version	Mean	$M_1 = 20.25$	$M_3 = 28.1$
	Standard Deviation	5.85	2.16
	Range	20	11
L_2 Version	Mean	$M_2 = 13.65$	$M_4 = 23.2$
	Standard Deviation	5.94	4.65
	Range	23	15

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